Some Insights into the Method of Center Projection*

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We present several new results which pertain to the successes of center projection in maximal center gauge (MCG). In particular, we show why any center vortex, inserted "by hand" into a thermalized lattice configuration, will be among the set of vortices found by the center projection procedure. We show that this "vortex-finding property" is lost when gauge-field configurations are fixed to Landau gauge prior to the maximal center gauge fixing; this fact accounts for the loss of center dominance in the corresponding projected configurations. Variants of maximal center (adjoint Landau) gauge are proposed which correctly identify relevant center vortices.

1. CENTER PROJECTION IN MAXIMAL CENTER GAUGE

In recent years a wealth of evidence has been accumulated on the lattice in favour of the center vortex theory of colour confinement. Our procedure [1] for identifying center vortices consists of the following steps:

1. Generate thermalized SU(2) lattice gauge field configurations.

2. Fix to *maximal center gauge* by maximizing:

$$\mathcal{R}[U] = \sum_{x,\mu} \Big|\mathrm{Tr}[U_\mu(x)]\Big|^2 . \qquad (1)$$

This in fact is *adjoint Landau gauge*; the above condition is equivalent to maximizing

$$\mathcal{R}[U^A] = \sum_{x,\mu} \mathrm{Tr}[U^A_\mu(x)] . \qquad (2)$$

3. Make *center projection* by replacing:

$$U_\mu(x) \to Z_\mu(x) \equiv \mathrm{signTr}[U_\mu(x)] . \qquad (3)$$

4. Identify excitations (*P-vortices*) of the resulting Z_2 lattice configurations.

P-vortices after center projection in MCG appear to be correlated with thick center vortices of full configurations [1,2]. Their density scales in MCG [3]. Removal of center vortices destroys confinement and restores chiral symmetry [4].

In the present paper we address the question why the above procedure is able to locate center vortices and why it in some cases fails, on lattice configurations preconditioned in a special way.

2. VORTEX-FINDING PROPERTY

The simplest condition which a successful method for locating center vortices has to fulfill is to be able to find vortices inserted into a lattice configuration "by hand". This will be called the *"vortex-finding property"*. Does the method described in Section 1 have this property?

An argument for a positive answer is rather simple: A center vortex is created, in a configuration U, by making a discontinuous gauge transformation. Call the result U'. Apart from the vortex core, the corresponding link variables in the adjoint representation, U^A and U'^A, are gauge equivalent. Let $\mathcal{R}[U^A] = \max$ be a complete gauge-fixing condition (e.g. adjoint Landau gauge) on the adjoint links. Then (ignoring both Gribov copies and the core region) U^A and U'^A are mapped into the same gauge-fixed configuration $\tilde{U}^A$. The original fundamental link configurations U and U' are thus transformed by the gauge-fixing procedure into configurations $\tilde{U}$, $\tilde{U}'$ which correspond to the *same* $\tilde{U}^A$. This means that $\tilde{U}$, $\tilde{U}'$ can differ only by continuous or discontinuous Z_2 gauge transforma-

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tions, with the discontinuous transformation corresponding to the inserted center vortex in U'. Upon center projection, $\tilde{U},\ \tilde{U}' \to Z,\ Z'$, and the projected configurations $Z,\ Z'$ differ by the same discontinuous Z_2 transformation. The discontinuity shows up as an additional thin center vortex in Z', not present in Z, at the location of the vortex inserted by hand.

This vortex-finding property goes a long way towards explaining the success of maximal center gauge in locating center vortices in thermalized lattice configurations, and also suggests that there may be an infinite class of gauges with this property.

However, there are two caveats that could invalidate the argument:

1. We have neglected the vortex core region, where U and U' differ by more than a (dis)continuous gauge transformation; and

2. Fixing to $\mathcal{R}[U^A] = \max$ is bedeviled by Gribov copies.

To find out whether these problems destroy the vortex-finding property, we have carried out a series of numerical tests. The simplest is the following:

1. Take a set of equilibrium SU(2) configurations.

2. From each configuration make three:

I – the original one;

II – the original one with $U_4(x,y,z,t) \to (-\mathbf{1}) \times U_4(x,y,z,t)$ for $t = t_0$, $x_1 \le x \le x_2$ and all y, z, i.e. with 2 vortices (one lattice spacing thick) inserted by hand. To hide them a bit, a random gauge copy is made of the configuration with inserted vortices;

III – a random copy of I.

3. Measure:

$$G(x) = \frac{\sum_{y,z} < P_I(x,y,z) P_{II}(x,y,z) >}{\sum_{y,z} < P_I(x,y,z) P_{III}(x,y,z) >}\ . \tag{4}$$

$P_i(x,y,z)$ is the Polyakov line measured on the configuration $i =$I, II, or III.

If the method correctly identifies the inserted vortices, one simply expects

$$G(x) = \begin{cases} -1 & x \in [x_1, x_2] \\ \ \ 1 & \text{otherwise} \end{cases}\ . \tag{5}$$

The result of the test is shown in Fig. 1. The inserted vortices are clearly recognized, and the associated Dirac volume is found in its correct location.

A more sophisticated test is to insert vortices with a core a few lattice spacings thick. Our method also passes that test satisfactorily.

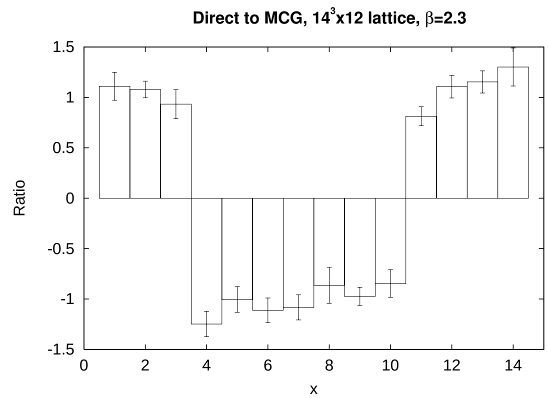

Figure 1. Graph of $G(x)$ for configurations with thin inserted vortices. Configurations are fixed directly to the maximal center gauge. The discontinuity was inserted to the time links within the volume $4 \le x < 11,\ 0 \le y < 14,\ 0 \le z < 14$ at the time slice $t = 0$.

3. WHEN GRIBOV COPIES BECOME PROBLEMATIC: PRECONDITIONING WITH LANDAU GAUGE

Gribov copies in maximal center gauge do not seem to be a severe problem in our procedure; it appears that P-vortex locations vary comparatively little, from copy to copy [1].

However, it has been shown recently [5] that if one first fixes to Landau gauge (LG), before relaxation to maximal center gauge, center dominance is lost.

This failure has a simple explanation: LG preconditioning destroys the vortex-finding property. This is illustrated by redoing the test shown in Fig. 1, only with a prior fixing to Landau gauge. The result, shown in Fig. 2, is that the vortex-finding condition is not satisfied; the Dirac volume is not reliably identified.

The Gribov copy problem, which is fairly harmless on most of the gauge orbit, seems severe enough to ruin vortex-finding on a tiny region of the gauge orbit near Landau gauge.[1]

[1] Cooling and smoothing, which modify thermalized configurations and greatly expand vortex cores, also pose some problems for center projection. Whether these are related to the Gribov problem, as found in Landau gauge preconditioning, is currently under investigation.

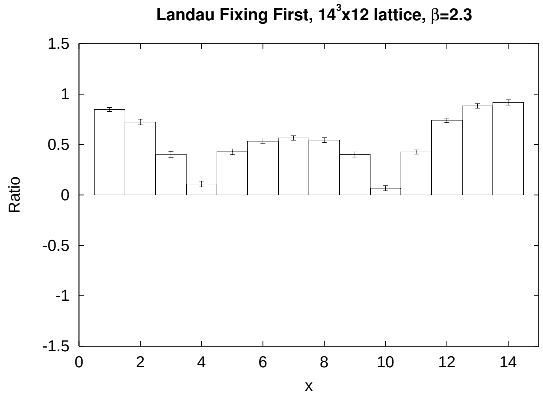

Figure 2. Graph of $G(x)$ for configurations with thin inserted vortices. Configurations are first fixed to the Landau gauge, and only then to MCG.

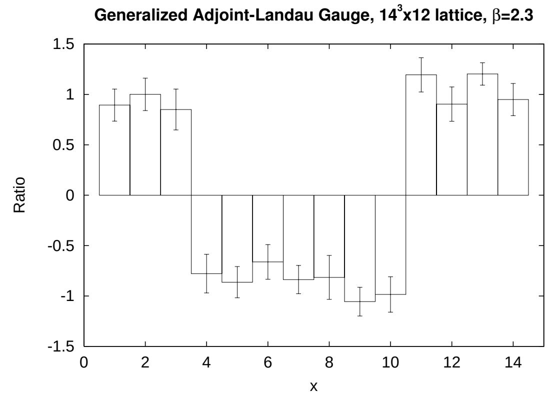

Figure 3. Graph of $G(x)$ for configurations with thin inserted vortices. Configurations are fixed to the gauge maximizing (6) with $c_\mu = \{1, 1.5, 0.75, 1\}$.

4. MCG IS NOT ALONE

The vortex-finding argument above does not seem to single out MCG. In fact, there should exist (infinitely) many gauges with the vortex-finding property. They should fulfill the following conditions:

1. The gauge fixing condition depends on the *adjoint* link variable.

2. The gauge fixing condition is complete for adjoint links, leaving a residual Z_2 gauge symmetry for fundamental links.

3. The gauge fixing condition is smooth, the gauge-fixed adjoint link is close to the identity matrix for large β.

An example is a slight generalization of MCG, namely a gauge maximizing the quantity

$$\mathcal{R}'[U] = \sum_{x,\mu} c_\mu \left|\mathrm{Tr}[U_\mu(x)]\right|^2 \qquad (6)$$

with some choice of c_μ, e.g. $c_\mu = \{1, 1.5, 0.75, 1\}$. Fig. 3 shows that this gauge has the vortex-finding property. Also, center dominance is observed in this gauge, in the same manner as in MCG.

5. CONCLUSION

We conclude with a sort of tautology: *To find center vortices, one must use a procedure with the vortex-finding property.* If that property is destroyed somehow, e.g. by Landau gauge preconditioning, then center vortices are not correctly identified, and center dominance in the projected configurations is lost. This fact does not call into question the physical relevance of P-vortices found by our usual method (which *has* the vortex-finding property); that relevance is well-established by the strong correlation that exists between these objects and gauge-invariant observables.

A gauge-fixing technique which completely avoids the Gribov copy problem is desirable. A viable alternative has been proposed by Ph. de Forcrand at this conference [6].

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